The Internal Consistency of Simultaneous Heat and Mass Transfer Relationships

DONALD R. OLANDER
University of California, Berkeley, California

In illustrating some of the fundamental difficulties encountered in the analysis of chemical engineering problems, White and Churchill (*1*) discussed an apparent inconsistency in the treatment of simultaneous heat and mass transfer in a vapor-liquid dehumidifier. The anomaly, they claimed, was due to the fundamental incompatibility of potential-difference and potential-gradient models in describing the rate of heat and mass transport. In this brief note an attempt will be made to show that the inconsistency stems from a much more prosaic source, namely that some of the relationships involved are approximations. The offending equations however are enthalpy balances, which care not a whit whether the heat fluxes therein are expressed as functions of differences or gradients of temperature; one need only retain these heat-flux terms as unspecified q's to prove that the inconsistency remains. Specifically, Equations (26) and (27) of reference *1* each lack one term which, because of its relatively small magnitude, has been universally disregarded. Naturally, the neglected terms become apparent when the internal consistency of the entire set of equations is examined.

It was pointed out that the equations

$$dL = GdH \qquad (1)$$

$$GdH = -N_A a dZ^* \qquad (2)$$

$$-Gs\,dt = h_v a\,(t - t_i)\,dZ \qquad (3)$$

$$LC_l d\,t_l = h_l a\,(t_l - t_i)\,dZ \qquad (4)$$

$$h_v a(t - t_i) + h_l a(t_l - t_i) = -N_A a\,\lambda_i \qquad (5)$$

can be appropriately combined to yield

$$G\,[s\,d\,t + \lambda_i\,dH] = LC_l d\,t_l \qquad (6)$$

Equation (6) was then compared to the differential enthalpy balance over both phases, which, with the thermodynamic identity,

$$\lambda_{t_1} + C_l\,(t_1 - t_2) = \lambda_{t_2} + C_v\,(t_1 - t_2) \qquad (7)$$

can be written as

$$G\,\{s\,d\,t + [\lambda_t + C_l(t - t_l)]dH\} = L\,C_l\,d\,t_l \qquad (8)$$

The authors indicated that Equations (6) and (8) could be combined to yield results which suggested the impossibility of operating a cooling tower with hot water and cold dry gas. The reason given for this anomaly indicated that Equations (3) through (5) were incompatible and that Equations (3) and (4) did not adequately represent the sensible heat fluxes required by the enthalpy balance.

It is a great relief to find that, after years of misuse, the logically misleading approximations to the gas- and liquid-phase differential enthalpy balances [Equations (3) and (4)] and the equally meaningless flux balance around the interface [Equation (5)] have finally been exposed. As a further impetus to the speedy demise of these relations the present author would like to point out the apparent source of the difficulty.

Equations (1), (2), and (8) are perfectly straightforward expressions of conservation of mass and the first law of thermodynamics. The prime error in attempting to combine these relations with Equations (3), (4), and (5) is that one too many restrictions have been imposed upon the system; that is, if sensible heat fluxes are defined by Equations (3) and (4) (whether these definitions are useful and/or well-founded is another matter), then we are not at liberty to specify the relationship between these fluxes, as Equation (5) assumes. The so-called "flux balance at the interface" is not an independent relation but is an auxiliary expression derivable from Equations (2) through (4) and (8). Thus if we consider the definitions of Equations (3) and (4), then Equation (5) must be of the form

$$h_v a\,(t - t_i) + h_l a(t_l - t_i) = -N_A a[\lambda_t + C_l(t - t_l)] \qquad (9)$$

Nevertheless both Equations (5) and (9) are logically superfluous statements of the behavior of the system. Moreover Equation (5) is incorrect.

Next we might inquire as to the origin of Equations (3) and (4). From what basic principle were they derived? Are they exact statements of that principle or convenient approximations?

Consider a differential slice of a dehumidifier, as sketched in Figure 1.

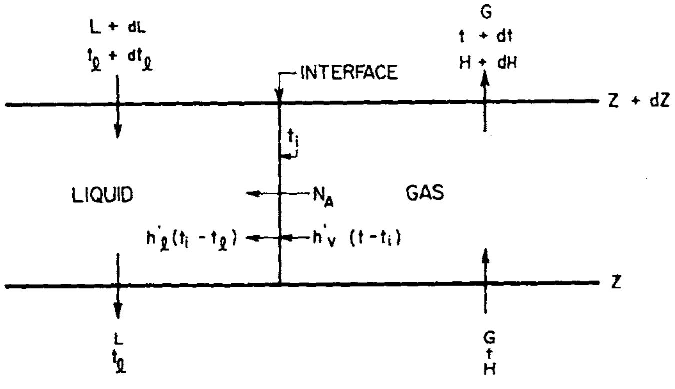

Fig. 1. Flows, compositions, and temperatures in a differential slice of a contactor undergoing simultaneous heat and mass transfer.

---

* $N_A = k_g M_v(p - p_i)$. Note the inversion of p and p_i compared with Equation (23) of reference *1*.

An enthalpy balance over the vapor phase up to but not including the interface proper would contain the following terms:

$$\text{Rate of enthalpy in} = G\overline{C}_{NC}(t - t_r) + GH[C_v(t - t_r) + \lambda_r]$$

The rate of enthalpy removal must contain the contribution of the diffusing vapor, which leaves the control volume at a rate $N_A a\, dZ$ (per unit cross section of tower) at a temperature t_i:

$$\text{Rate of enthalpy out} = G\overline{C}_{NC}(t + dt - t_r) + G(H + dH)[C_v(t + dt - t_r) + \lambda_r] + N_A a\, dZ\,[C_v(t_i - t_r) + \lambda_r]$$

The difference in these enthalpy rates is properly attributed to a sensible heat flux from the vapor phase toward the liquid. With Equation (2) there results

$$h'_v a\,(t - t_i)dZ = -Gs\,dt - G\,dH\,C_v\,(t - t_i) \qquad (10)$$

Similarly, an enthalpy balance over the liquid phase up to but not including the interface yields

$$h'_l a\,(t_i - t_l)dZ = LC_l dt_l - G\,dH\,C_l\,(t_i - t_l) \qquad (11)$$

In Equation (11) the contribution of the diffusive flux entering the control volume as a liquid at temperature t_i has been included.

Returning to Equations (3) and (4) one can see that these relations differ from the enthalpy balances just considered in their omission of the right-hand term of Equations (10) and (11). Equations (3) and (4) are therefore approximations of exact enthalpy balances which have been in use so long that they have acquired the aura of fundamental relationships. By their inexact nature they can only lead to anomalies when the internal consistency of the entire set of equations describing the simultaneous heat and mass transfer in the dehumidifier is considered. Their usefulness in the practical task of analyzing this type of contactor depends on the magnitude of the omitted term. The heat transfer coefficients h'_v and h'_l are more nearly equal to those obtainable from correlations of heat transfer experiments conducted in the absence of simultaneous mass transfer [appropriately corrected by the method of Ackermann (2)]. This is generally not true of h_v and h_l.

It can easily be demonstrated that combining Equations (2), (8), (10), and (11) will lead to a "flux balance at the interface" given identically by Equation (5). Nevertheless the flux balance at the interface is still a superfluous concept. The proper enthalpy and material balances are the necessary and sufficient relations for a complete and internally consistent description of the process.

NOTATION

a = interfacial area for both heat and mass transfer per unit volume, sq.ft./cu.ft.
C_l = liquid heat capacity, B.t.u./(lb.)(°F.)
C_v = heat capacity of the condensible vapor, B.t.u./(lb.)(°F.)
$\overline{C}_{NC}$ = average heat capacity of the carrier gas, B.t.u./(lb.)(°F.)
G = mass velocity of carrier gas. lb./(hr.)(sq.ft.)
H = humidity, lb. vapor/lb.carrier gas
h_l, h'_l = liquid-phase heat transfer coefficient defined by Equations (4) and (11) respectively, B.t.u./(hr.)(sq.ft.)(°F.)
h_v, h'_v = gas-phase heat transfer coefficients, defined by Equations (3) and (10) respectively, B.t.u./(hr.)(sq.ft.)(°F.)
k_g = gas-phase mass transfer coefficient, lb. mole/(hr.)(sq.-ft.)(atm.)
L = mass velocity of liquid, lb./(hr.)(sq.ft.)
M_v = molecular weight of the condensible vapor, lb./lb.mole
N_A = rate of mass transfer, lb./(hr.)(sq.ft.)
p, p_i = partial pressure of the condensible vapor in the gas phase and at the interface respectively, atm.
s = humid heat, equal to $\overline{C}_{NC} + HC_v$, B.t.u./(lb.)(°F.)
t = gas-phase temperature, °F.
t_l = liquid-phase temperature, °F.
t_i = temperature of interface, °F.
t_r = reference temperature for enthalpy balance, °F.
Z = height, ft.
λ = heat of vaporization, B.t.u./lb.

LITERATURE CITED

1. White, R. R., and S. W. Churchill, *A.I.Ch.E. Journal*, 5, No. 3, 354 (1959).
2. Ackermann, G., *Forschungsheft*, 382, 1 (1937).

Effectiveness of Mixing Tanks in Smoothing Cyclic Fluctuations

EDGAR B. GUTOFF

Brown Company, Berlin, New Hampshire

Mixing tanks are useful in smoothing out fluctuations in continuous process streams. If perfect mixing is assumed in the mixing tank, calculation of the effectiveness of the tank in reducing cyclic fluctuations can be fairly straightforward, as was shown by Danckwerts and Sellers (*4*), Kramers and Alberda (*8*), Gutoff (*6*), and Walker and Cholette (*13*).

When, on the other hand, the assumption of perfect mixing is not justified, it is extremely difficult to calculate the mixing tank effectiveness, for the fluid regime is very complex. [Danckwerts (*2*, *3*) describes how experimental data on a given tank can be used for this calculation.] Cholette and Cloutier (*1*) assumed that in part of the tank volume perfect mixing takes place, that part is stagnant, and that some plug flow and short circuiting of feed occur. However it is difficult to calculate beforehand what the magnitude of these factors will be. The model presented below is believed to be more fundamental in that it takes into account the behavior of the agitator.

Edgar B. Gutoff is with Ionics, Incorporated, Cambridge, Massachusetts.